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COMPUTATIONAL STUDY OF UNSTEADY VISCOUS FLOW IN FLEXIBLE VESSELS

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ABSTRACT

In this paper, we investigate unsteady behavior of flexible vessels carrying a blood flow. Membrane model with constant tension for the vessel walls and incompressible newtonian fluid approximation for blood is adopted. Developed computational model is applied to simulate the coupled fluid-wall behavior in 2D collapsible channels and 3D collapsible tubes.

INTRODUCTION

Vessels carrying blood flow in a human body are known to be flexible tissues. Interaction of the internal blood flow with the vessel wall compliance, in addition to significant alteration of the fluid mechanical properties (such as shear and normal stresses) with respect to rigid wall cases, can also result in a variety of interesting mechanical phenomena, such as flow limitation, self-exciting oscillations (flutter), or tube collapse. These phenomena are especially pronounced at higher Reynolds number and thus are relevant to the medical condition of stenosis caused by atherosclerosis, which results in higher local flow rates and elevated risk of collapse manifestation. In the current paper, we investigate the flutter and collapse phenomena in application to 2D collapsible channels and 3D collapsible tubes. We model the elastic vessel wall as a biological membrane with the constant tension and no bending stiffness, supported by a common assumption of negligible bending stiffness in biological materials [1, 2]. We stress, however, that the tube law and the flow limitation regime depend strongly on the elasticity model. Thus, bending rigidity would act to reduce the wall collapse [3] and postpone the onset of divergence and flutter.

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NUMERICAL METHOD

Numerical method consists of an Arbitrary Lagrangian-Eulerian (ALE) formulation of incompressible Navier-Stokes equations coupled to a simple constant-tension geometrically nonlinear elastic wall model by the kinematic and traction boundary conditions:

$$\mathbf{u} = \mathbf{w}, \quad (1)$$

$$\sigma_n - p_e = -T\gamma, \quad (2)$$

where $\mathbf{u}$ is the fluid velocity, $\mathbf{w}$ is the wall velocity, σ_n is the normal fluid stress acting on the membrane, p_e is the external pressure, T is the longitudinal tension and γ is the nonlinear wall curvature (mean curvature in a 3D case). The coupled unsteady fluid-structure problem is solved numerically with the spectral element method [4], which is analogous to the finite element method, but it retains a specified number of collocation Gauss-Lobatto points within each element, thus giving it spectral convergence properties.

COLLAPSIBLE CHANNELS

For validation of the developed computational method, we first simulate the steady and unsteady results of Luo&Pedley [5, 6] for the flow in a 2D collapsible channel. We find excellent agreement with Luo&Pedley in calculated steady wall shapes and pressure profiles for several cases. For every Reynolds number, if tension is reduced below some critical value T_u, the steady solution becomes unstable, and self-exciting oscillations occur. Dependence of the critical tension T_u on Reynolds number is

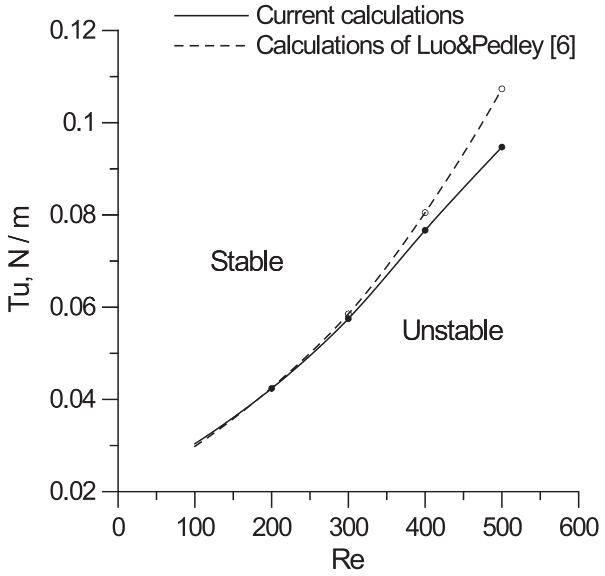

FIGURE 1. DEPENDENCE OF THE CRITICAL TENSION T_u ON REYNOLDS NUMBER.

given in Fig. 1 for $L/D = 5$, $L_d/D = 30$ and $p_e - p_{out} = 1.75Pa$ (L is the length of the elastic segment, L_d is the length of the downstream rigid segment). It is believed that the onset of self-exciting oscillations is related to the flow limitation and the subsequent regime of negative dependence of the flow rate Q on the pressure drop $p_{in} - p_{out}$, which makes a system statically unstable [7]. If the tension is reduced even further, below the second critical tension T_c, complete wall collapse takes place and numerical solution breaks down. This occurs because the tube law describing the elastic response of the wall as an area change versus the applied pressure difference $\Delta p = p_e - p$ allows for a complete collapse to a zero area in a constant-tension model under a critical value $\Delta p/T_c = 2/((L/2)^2 + 1)$ (which can be easily deduced from the traction condition (2) in the absence of flow). With the bending rigidity included, the critical value of $\Delta p/T_c$ increases significantly, and the complete wall collapse would occur under much higher pressure difference or much lower tension.

COLLAPSIBLE TUBES

If the same constant-tension model is applied in 3D case, one can see that axisymmetric steady solutions for a wall shape in the absence of flow, giving the tube law, do not exist for the elastic segments of length $L > 1.32$. Indeed, say, for a zero transmural pressure $\Delta p = 0$, the corresponding axisymmetric solution would involve minimal surface $r = c\cosh(x/c)$, with no real value for a constant c for $L > 1.32$. As a consequence, static divergence resulting in a complete collapse of axisymmetric flow solution is observed for any values of tension and any positive pressure difference $\Delta p = p_e - p$ for elastic membranes longer than $L = 1.32$. It was confirmed that shorter membranes do possess steady axisymmetric solutions. Self-exciting oscillations have not been observed in the axisymmetric cases for neither short nor long membranes. We therefore conclude that, in accordance with the recent findings of Heil&Boyle [8], self-exciting oscillations in tubes can only arise from non-axisymmetricaly buckled initial configurations. Whether such oscillations can be observed with a simple constant-tension membrane model is currently under investigation.

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